During School Year (SY) 2013-14, Fairfax County Public Schools (FCPS) undertook a study of the content and sequence of its mathematics curriculum, as well as its acceleration and placement practices for students. This study came about as the result of a School Board discussion about the impact mathematics coursework sequencing and acceleration of mathematics instruction has on student mastery of higher-level mathematics content. The School Board directed the division to undertake an examination of issues related to these concerns: “Evaluate the Division’s current mathematics course sequencing and other mathematics programs for effectiveness and prepare a report for the Board.” Based on this request, the Office of Program Evaluation (OPE) designed a study with advice from both school and central office-based stakeholders.

The final report summarizes for executive staff a series of four data analysis reports (available at http://www.fcps.edu/pla/ope/tech_r/math_study.shtml) that discussed in depth the findings of the analyses conducted for this study. The four reports encompassed a literature review (report 1), analysis of student performance (report 2), expert and stakeholder views of the FCPS curriculum (report 3), and an analysis of how school practices relate to student performance (report 4). This report summarizes key findings across all four reports, discusses implications for FCPS, and provides recommendations to the Superintendent.

**Organization of This Report**

The report is organized into a background section, three data sections, and two concluding sections – one focused on implications, the other on recommendations:

- **Section 1: Data Sources** – Describes the sources of evidence used to investigate the mathematics curriculum.
- **Section 2: The FCPS Mathematics Curriculum** – Describes findings about the rigor and relevance of the FCPS written mathematics curriculum, as well as the sequencing of content.
- **Section 3: Student Opportunities for Enrichment and Acceleration** – Describes findings about FCPS student access to mathematics enrichment and acceleration, as well as associated student performance in mathematics.
- **Section 4: Student Opportunities and Performance** – Describes findings about FCPS student performance linked to the opportunities students received for enrichment and acceleration.
- **Section 5: Conclusions and Implications** – Summary and conclusions about the findings, highlighting areas in which FCPS is functioning well (which require no action) and areas that remain a challenge (which require action).
- **Section 6: Recommendations** – Provides OPE recommendations to the Superintendent.
Section 1: Data Sources

This report relies primarily on the FCPS high school mathematics curriculum, as well as enrollment and test data from all FCPS students taking high school courses during SY 2012-13. While investigation of the curriculum was primarily at the high school level, to match the issues that surfaced during the School Board conversation, the study also investigated performance of elementary and middle school students, as well as opportunities for mathematics classes and courses available at those grade levels. Additionally, information from a sample of 41 FCPS elementary, middle, and high schools, teacher focus groups, and surveys from teachers, students, and parents at these same schools supplements the curricular and student performance evidence. The sample of schools was selected to represent the entire school division and allow for generalizing about FCPS as a whole. The specific study questions are available in the Math Study Design. Additional details about the curriculum documents, sampling, survey, and other methodology are available as Appendix A. Terms used in this report, as well as in analytical reports one through four, are available in Appendix B.

Section 2: the FCPS Mathematics Curriculum

This study sought first to establish whether the content of the FCPS mathematics curriculum (commonly known as the FCPS Program of Studies [POS] for mathematics) was relevant, rigorous, and appropriately sequenced. An expert comparison by outside consultants, Curriculum Management Systems, Inc. (CMSi¹), comparing the FCPS curriculum to external content standards, as well as data collected from parents, students, and teachers, were analyzed to draw conclusions about these three dimensions of the curriculum.

The FCPS written curriculum, or POS, serves as a detailed roadmap of the content (e.g., facts, concepts, laws, rules, skills, etc.) that teachers deliver while instructing students and, thus, forms the foundation for student performance. That is, strong student performance across the school division requires a strong written curriculum that is relevant, rigorous, and well-sequenced in support of expected student performance. While the study of instruction was considered beyond the scope of this study, it is recognized that curriculum defines what students should learn while instruction defines how students learn it. The two work in tandem to generate student learning.

This section of the report provides information about three dimensions of the FCPS written mathematics curriculum: (a) relevance, (b) rigor, and (c) sequence. For this study relevance was defined as content related to students' lives, development of 21st century skills, and adequate preparation for post-secondary education and the world of work. Rigor was defined as the amount of academic (cognitive) demand designed into the written curriculum. Demand can span a range from low complexity (students

¹ CMSi (Curriculum Management Systems, Inc.) is a nationally recognized corporation that provides curriculum auditing™, a process for school system improvement developed by Fenwick English. Curriculum auditing, a useful process for school systems seeking improvement of the quality of design and delivery of curriculum in their schools, is available for school systems through Phi Delta Kappa International.
mainly recalling and recognizing previously learned concepts and principles) to high complexity (students mainly using abstract reasoning, planning, analysis, judgment and creative thought). Lastly, sequence was defined as the progression of content within and across grades or courses.

Finding 1: Both experts and stakeholders agreed that the FCPS mathematics curriculum covers relevant content. The experts’ conclusions stemmed from both the similarity of the FCPS high school curriculum to highly-regarded national and international curricula, as well as the curriculum’s inclusion of content that reflects college and career readiness standards.

External curriculum experts, CMSi, concluded that the FCPS high school mathematics curriculum is a highly relevant pursuit of study. This conclusion was based on a comparison of the FCPS curriculum with respected national (Common Core) and international (Finland) curricula. That is, the experts concluded that the topics covered by the FCPS mathematics curriculum match topics covered in curricula esteemed for reflecting relevant content. For example, the experts found that over 90 percent of the FCPS Standard and Honors Algebra II and Precalculus content matches or exceeds the national content standards of the Common Core and Finland’s curriculum.

Furthermore, the experts concluded that the FCPS high school curriculum reflects relevant College and Career Readiness Standards, another measure of the relevance of the FCPS curricular content. CMSi’s finding about College and Career Readiness Standards was based on the match between the FCPS curriculum and scores on the ACT. From middle school through Algebra II, the FCPS curriculum covered more than 90 percent of the ACT Standards required for college admission (ACT scores of 20 to 32). Only at the highest scores (33 to 36) did the FCPS curriculum align less closely (39 percent) to ACT Standards. ACT Standards found at this score level that are not covered by the FCPS curriculum require students to connect and integrate knowledge across courses.

Lastly, the majority of critical FCPS stakeholders perceived the FCPS mathematics curriculum as containing relevant content. Parents (83 percent), students (90 percent), and teachers (84 percent) all perceived FCPS mathematics content to be relevant and important for students’ future success. [For additional details see Section 2 of Math Study Report 3.]

Finding 2: Overall, the FCPS high school mathematics curriculum was judged by CMSi as similar in rigor to highly-regarded mathematics curricula. CMSi also concluded that the FCPS curriculum has room for improvement to insure higher levels of rigor for all students.

CMSi concluded that the FCPS high school mathematics curriculum is a rigorous pursuit of study based on a comparison with respected national and international curricula. That is, the level of difficulty in the FCPS curriculum was judged as comparable to that found in the other two curricula (Common Core, Finland). More specifically, the majority of the FCPS Algebra II and Precalculus curricula aligns or exceeds the academic (cognitive) demand found in the national standards of the Common Core (83-89 percent) and aligns or exceeds the international content standards of Finland’s mathematics curriculum (69-73 percent).
Although aligned in rigor to these other two curricula, the experts also determined that the FCPS mathematics curriculum is too limited in its emphasis on building conceptual knowledge\(^2\) and application of mathematics, a common challenge for math curricula. Of particular note, the experts determined that standards reflecting conceptual knowledge and application are found primarily in the FCPS extended curriculum (which is received mostly by students taking Advanced or Honors Math). However, CMSi’s observation was not reflected in the perceptions of the majority of students, parents, and teachers who reported that FCPS students experience appropriate levels of challenge in their mathematics coursework (an indirect measure of the curriculum’s rigor). The student, parent and teacher views of the curriculum may reflect instruction provided to students more so than the written curriculum. Thus, it may be that FCPS teachers have already embedded the research-based mathematical practices into their instruction. The experts encouraged FCPS to expand the practices within the written curriculum to support consistent instruction in conceptual knowledge and application of mathematics across all FCPS classrooms. [For additional details see Section 3 of Math Study Report 3.]

Finding 3: CMSi indicated that the mathematics curriculum is well-sequenced, meaning it presents content in an order that provides students with exposure to concepts that build upon one another in a logical and understandable way.

In considering the curricular sequence (i.e., the order in which topics are covered by the FCPS curriculum within and across years), CMSi concluded that the full set of the FCPS elementary and middle school curriculum should adequately prepare students for success in high school mathematics (including courses at TJHSST). Furthermore, the majority of parents, students and teachers agreed that the mathematics curriculum is well-sequenced. A slightly larger percentage of teachers (79 percent) than parents (69 percent) and students (69 percent) perceived students as having been placed correctly according to the student’s ability and at a level in which the student was appropriately challenged. [For additional details see Section 4 of Math Study Report 3.]

Finding 4: The overall strong sequencing of the FCPS curriculum is threatened by FCPS acceleration practices that create gaps in sequencing.

Most pathways to acceleration in elementary and middle school result in gaps in curricular sequencing unless teachers backfill missed curriculum from the prior course. For example, when students move from Regular Math in Grade 6, to Honors Math in Grade 7, they skip over the Grade 7 mathematics standards because Honors Math 7 covers the Grade 8 mathematics curriculum. At the elementary level, teachers of students receiving acceleration (i.e., Advanced Math) are directed to backfill “missed” standards from the prior grade. At the middle school level, no specific direction is provided, so filling in gaps would fall under the general heading of differentiated instruction. At all grade levels, however, it falls to teachers to determine which gaps need to be filled when students accelerate rather than having

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\(^2\) Knowing **why** something happens. Conceptual knowledge refers to understanding of the principles, rules, and generalizations that govern mathematics and the interrelationships between them. For example, understanding why adding two positive numbers should always yield a value that is larger than either of the original numbers would reflect conceptual knowledge in mathematics.
it specified in the written curriculum. In focus groups, teachers noted that backfilling of instruction on missed standards is handled inconsistently and is difficult to fit in, given the other demands of the written curriculum. [For additional details see Section 4 of Math Study Report 3.]

Finding 5: CMSi expressed concerns about the redundancy in the content of the Precalculus, AP Calculus AB and AP Calculus BC curricula at TJHSST.

In addition to a general concern about the FCPS curriculum, the School Board expressed a specific interest in AP Calculus AB course-taking at TJHSST. Expert review of the PreCalculus, AP Calculus AB, AP Calculus BC sequence at TJHSST determined redundancy in the curriculum. Specifically, all topics in AP Calculus AB were judged to be covered by the Precalculus and AP Calculus BC courses, rendering the AP Calculus AB course unnecessary. Investigation of how many students at TJHSST may be taking this redundant sequence found that the majority of students enrolled in AP Calculus BC did so without having taken AP Calculus AB. The study also determined that a larger proportion of AP Calculus BC students at TJHSST had previously taken AP Calculus AB than at other FCPS high schools (29 percent at TJHSST vs. 18 percent at other FCPS high schools). Thus, statistically, there was a small effect among TJHSST students in comparison to other FCPS students in choosing this redundant pathway. Further investigation into this phenomenon could determine why more TJHSST students are choosing to take the redundant AP Calculus AB course. [For additional details on the TJHSST curriculum see Section 5 of Math Study Report 3; for additional details on AP Calculus course taking see Section 4 of Math Study Report 4.]

Section 2 Summary: Generally the FCPS written curriculum was judged by both experts and local stakeholders (students, parents, and teachers) as reflecting appropriate relevance and rigor. Experts concluded that FCPS should provide more written guidance within the curriculum to stress mathematical practices that enhance conceptual knowledge and application. Furthermore, both experts and stakeholders judged the curriculum as well-sequenced in providing the foundational knowledge necessary to support success in high school coursework, including more rigorous coursework available to students at TJHSST. Specific analyses examining the redundancy of Precalculus, AP Calculus AB and AP Calculus BC coursework at TJHSST, indicated that the AP Calculus AB course provides no new content and TJHSST students are more likely to enroll in this redundant course than students at other FCPS high schools who go on to take AP Calculus BC.

Section 3: Student Opportunities for Enrichment and Acceleration

In addition to investigating the written curriculum that FCPS uses for mathematics instruction, this study also investigated the opportunities that FCPS students receive for exposure to enriched curriculum. That is, the FCPS curriculum divides standards into those that all students should learn and those that should be provided to advanced learners. The study, therefore, included an examination of the procedures used to place students into mathematics classes and courses, especially those that provide enriched mathematics content (such as Advanced Mathematics in elementary school and honors courses in middle and high school). Additionally, the study considered students’ opportunities to
accelerate into higher levels of mathematics (e.g., skipping a grade level in mathematics during elementary school, taking high school mathematics courses during middle school).

Finding 6: Decisions about who receives enriched and accelerated content differ in elementary versus middle and high school. These differences potentially result in students, especially at the elementary level, receiving more or less rigorous content than their quantitative aptitude\(^3\) suggests they can handle.

At elementary school, school staffs primarily determine opportunities to receive enriched or accelerated content. Quantitative aptitude appears to play less of a role in determining whether a student is placed in Advanced Math (which accelerates students one grade and where enriched content is intended to be included routinely) than may be appropriate. For example, approximately one-third of students identified as high in quantitative aptitude do not receive Advanced Math during elementary school. In addition, among students assessed as average in quantitative aptitude, quantitative aptitude only partially explains which students are offered Advanced Math. In other words, students with the same levels of mathematics aptitude are not consistently offered the same opportunities to access higher levels of rigor. Thus, it appears that elementary schools frequently rely on factors beyond student aptitude for advanced content when determining who should accelerate or access the most rigorous elementary mathematics curriculum offered by the County.

The elementary placement approach contrasts with that of the secondary (middle, high) level where students and parents are more frequently determining whether students access the rigorous curriculum via honors classes. In middle and high school, where FCPS Regulation 3250 requires open enrollment practices, student preference and parent input play a greater role. Nonetheless, schools at this level still report using many of the same criteria as elementary schools to determine student placement, including past SOL performance and teacher recommendations. In fact, focus group evidence as well as a review of documents provided to students and parents about mathematics course-taking indicate that some FCPS middle and high schools have instituted processes that require students and parents to sign “waivers” when they choose to override a school’s recommended mathematics placement.

Survey evidence indicated that stakeholders differ in their satisfaction with placement practices in ways that align with whether they have primary control over decision making. Elementary teachers (who have more control over placement decisions) reported greater satisfaction with student placement than their middle and high school counterparts. The converse was true for students and parents; middle and high school students and parents reported greater satisfaction with placement than elementary students and parents. [For additional details see Section 4 of Math Study Report 2.]

\(^3\) As measured by the CogAT, which categorizes student scores into significantly above average, above average, average, below average, and significantly below average on quantitative aptitude. The CogAT is one of several measures used to determine eligibility for FCPS’ Advanced Academic services in elementary school.
Finding 7: Schools differ in the amount of enriched content provided to students. At the elementary level, there is a lot of variance between schools in what students receive. At the middle and high school levels there is more consistency in what students receive. These differences, especially at the elementary level, potentially result in students at different schools having different opportunities to engage with the most rigorous content.

FCPS schools are less consistent at the elementary level than at the middle and high school level in their practices around how much enriched content students receive. At the middle and high school level, courses typically determine the level of exposure to enriched content. FCPS middle and high schools report that most enriched standards are covered in honors courses and few, if any, are covered in standard level courses. In contrast, FCPS elementary schools reported a greater variety in the amount of enriched content provided to students, dependent on type of school (i.e., AAP center school, school offering local level IV services, school not offering level IV services) and type of student (i.e., general education student, student identified for Level IV services). For example, while central office has designed Advanced Mathematics to cover all extended indicators, schools reported covering anywhere from some to most to all of the enriched content with students. While on average more enriched content is typically provided to students taking Advanced Mathematics than those taking Regular Grade-level Mathematics, an individual elementary school might offer more of the enriched content to students taking Grade-level Math than to those taking Advanced Math. [For additional details see Section 4 of Math Study Report 2.]

Finding 8: Participation in Advanced Math during elementary school is a “gateway” to enriched and accelerated curriculum in middle and high school.

Advanced Math placement in elementary school leads to accelerated and/or Honors course taking in middle school (which, as described above, routinely includes more enriched content), regardless of math aptitude. For example, in one recent cohort of students, 98 percent of students with average quantitative ability who had participated in Advanced Mathematics during elementary school had completed a high school mathematics course by the end of middle school, while 58 percent of students with similar aptitude who took regular grade-level mathematics in elementary school had done so. Furthermore, middle school mathematics course-taking determines high school course-taking patterns since the majority of students who complete all three required high school mathematics courses by the end of tenth grade continue to take additional classes (typically AP, IB or above) in eleventh and twelfth grades. Thus, a student’s mathematics placement during elementary school, which determines early exposure to enrichment and acceleration, plays a critical role in that student’s high school course-taking. [For additional details see Section 3 of Math Study Report 4.]
Finding 9: Majorities of FCPS parents and students report expectations for high school mathematics course-taking that require acceleration.

A majority of both students and parents reported aiming for mathematics course completion beyond what is required by the state and FCPS requirements\(^4\) for graduation. Sixty percent or more of FCPS parents and students reported plans to complete AP/IB coursework or beyond prior to high school graduation. Based on these stakeholder expectations, acceleration is a necessity. That is, to reach these course levels, students have to complete Algebra I by eighth grade or, for most IB courses and courses beyond AP/IB, seventh grade. Moreover, the expectations are best supported by acceleration in elementary school via the Advanced Math program, which currently serves a minority of FCPS students. Parent and student expectations may explain why participation in accelerated mathematics coursework increases during middle school. Data indicate a majority of FCPS students enroll in at least one high school level mathematics course (Algebra I, Geometry, Algebra II) during middle school. [For additional details see Section 6 of Math Study Report 3.]

**Section 3 Summary:** Stakeholders report that, at the elementary level, school-based staff were the primary decision makers about which students were provided access to enriched and accelerated curriculum. In contrast, at middle and high schools the decision-making is primarily in the hands of parents and students. Schools differ in the level of exposure provided to students for enriched mathematics content. Elementary schools are less consistent than middle and high schools, possibly because they do not offer specific courses. Parent and student expectations for high school mathematics course-taking exceed graduation requirements and align with being accelerated by one year or more. The pattern for whether or not such expectations can be met is established in elementary school and is best supported by participation in Advanced Mathematics.

**Section 4: Student Opportunities and Performance**

Overall, the study has noted that mathematics performance in FCPS is relatively strong. [For additional details see Section 2 of Math Study Report 2.\(^3\)] Of particular interest is the impact of accelerated and enriched content on student performance. This issue in part reflects the School Board’s concern that students receiving accelerated mathematics content might be disadvantaged in some way. Thus, extending from placements with differing enriched content and acceleration, the study also investigated associated learning outcomes for students.

**Finding 10:** FCPS students exposed to an enriched or accelerated mathematics curriculum demonstrate higher mathematics achievement than those not exposed, regardless of quantitative aptitude.

The study determined that students who accelerated in elementary school (and also received enrichment) are doing well and do not seem to be detrimentally impacted in mathematics performance

\(^4\) For students who enter the ninth grade in SY 2013-14, to graduate with a standard diploma, students must complete three years of mathematics which includes a course beyond Geometry (which is most commonly Algebra II). For an advanced diploma, students must complete four years of mathematics and a course beyond Algebra II (which is most commonly Precalculus).
by the acceleration. And, the more students accelerate and are exposed to higher rigor, the better they perform. For example, based on the Spring 2013 SOL performance data, students who were accelerated in elementary school and took enriched high school coursework demonstrated a significant performance advantage over other students, even when controlling for prior achievement levels. For many of these students, this may not be surprising since those who accelerate often also have higher quantitative aptitude.

However, as reported earlier, students with high quantitative aptitude do not always receive exposure to advanced mathematics offerings, and students with average or even below average quantitative aptitude sometimes do receive such opportunities. Consistent with the notion that greater access to enriched and accelerated mathematics opportunities leads to higher performance, students of average or below average quantitative aptitude also rise to the challenge and are generally successful. For example, among students of average aptitude who had taken regular grade-level math during elementary school, 87 percent who enrolled in honors math for seventh grade passed the associated SOL test while 69 percent enrolled in regular seventh grade mathematics passed the associated SOL test. These findings match other studies that have shown that challenging students with more rigorous content typically leads to improved performance, even among lower achieving students. However, as reported in focus groups, teachers may find it overly burdensome to provide sufficient support for some of these students, who may arrive less prepared or less able to master the enriched content. [For additional details see Section 3 of Math Study Report 4.]

Finding 11: Students who receive grade-level instruction and little enriched content through elementary, middle and high school are challenged to meet mathematics standards for Algebra II.

Approximately one-third of FCPS students are enrolled in grade-level mathematics. These students cover a range of quantitative aptitude, from below average to average to above average. Similar to the findings reported above about the benefits of acceleration and enrichment, findings also indicate that students who neither accelerated in elementary school nor took higher rigor mathematics in high school (i.e., honors level) are having trouble performing successfully in Algebra II. For example, for one recent cohort of FCPS students, the average Algebra II SOL score for such students was 411, and, among most subgroups of students, fell into the failing range. In other words, current performance indicates that students receiving the FCPS regular mathematics curriculum throughout their schooling will be challenged to achieve proficiency on the Algebra II SOL test. [For additional details see Section 3 of Math Study Report 2.]

Finding 12: The highly accelerated students who receive the unique opportunities for rigor at TJHSST outperform similarly accelerated students attending other FCPS high schools.

In comparison to highly accelerated students at other FCPS high schools and while controlling for quantitative aptitude, attending TJHSST was associated with higher performance across all four measures of mathematics investigated (Algebra II SOL, AP/IB exams, PSAT, SAT/ACT). For example, when comparing the AP calculus performance of TJHSST students to other similar students in FCPS, TJHSST students typically attained a score of five on the AP Calculus BC exam (the highest possible
score), while students at other schools attained a score of three (the lowest score considered reflective of college level performance) on the same exam. However, as described above, TJHSST students were more likely to have completed the AP Calculus AB course prior to taking the AP Calculus BC exam than similar students at other schools, which was described by curriculum experts CMSi as a redundant course covering no new material.

Section 3 Summary: Overall, students who are offered the chance to take more rigorous or accelerated courses perform better, regardless of quantitative aptitude. Receiving the unique opportunities for rigor offered at TJHSST also led to better performance. However, the regular mathematics curriculum was shown to be inadequate preparation for high school coursework, especially for some subgroups of students.

Section 5: Conclusions and Implications

This section summarizes the findings and evidence described in this report. This section also provides implications for FCPS as the division seeks to maintain areas of excellence in mathematics curriculum and improve areas of concern.

FCPS' Mathematics Curriculum

Generally, external content experts and local students, parents, and teachers agree that the FCPS written mathematics curriculum, encompassing both expected and extended standards, represents relevant and rigorous mathematics standards. Furthermore, the curriculum is sequenced in a way that should support strong student learning. While experts suggested strengthening curriculum documents in a few areas to reflect mathematical emphases and practices (which could help with consistency from teacher to teacher or school to school and support greater conceptual understanding and application by students), the findings of this study indicate that the overall curriculum is not a source of concern. Of greater concern is how the curriculum is used and applied because some schools, particularly at the elementary level, reported using the enriched portions of the curriculum in ways that do not align with intentions (i.e., not providing all of the indicators to Advanced Mathematics students). This issue may be related to a level of awareness at some schools and disagreement with the intentions at other schools. Furthermore, although central office provides guidance that teachers should backfill curriculum standards for students who have accelerated past some parts of the curriculum, individual students may not be receiving the entire written curriculum.

Opportunities for Enrichment and Acceleration

The study revealed unintended variation in how schools make decisions about student placement into either more rigorous or accelerated (i.e., advanced) curricula. At the elementary school level, in particular, schools described different practices. Given the achievement data examined for this study, which clearly showed that involvement in elementary school advanced math set the pattern for later course-taking and performance, it appears critical to insure that as many students are accessing Advanced Mathematics as possible. Three findings appear to require corrective action. First, while the
evidence clearly shows the importance of exposing students to the highest mathematical rigor (i.e., the extended indicators), elementary schools are not universally providing them to Advanced Math students as intended. Second, there is a relatively sizeable group of students with above average quantitative skills who are not receiving Advanced Mathematics. Third, the finding that students who receive only the standard mathematics curriculum in high school are having problems passing Algebra II indicates that the standard curriculum is not sufficiently rigorous to carry students through coursework required for high school graduation.

**Productive Struggle**

Schools and teachers indicated practices and experiences that suggest discomfort with a majority of FCPS students participating in the County’s more rigorous and accelerated courses. However, these practices and experiences are not consistent with expectations held by parents and students regarding intended mathematics course taking. Overall, parents and students express a desire to have the majority of students participate in more rigorous and accelerated courses. There is now a body of literature that has introduced the importance of “productive struggle” in mathematics learning. This concept refers to “effort to make sense of mathematics, to figure something out that is not immediately apparent” (Heibert & Grouws, 2007). When engaged in productive struggle, students must grapple with concepts and come up with solutions, developing persistence and resilience in pursuing and attaining the learning goal or understanding (Inservice.ASCD.org, 2012). Thus, productive struggle leads to greater retention and deeper understanding of material due to effort put in by students. Students, parents, and teachers seem to be reflecting differing perspectives about the amount of struggle they consider acceptable.

**Section 6: Recommendations**

Based on the findings from this study the Office of Program Evaluation offers the following recommendations to the Superintendent.

**Recommendations to the Superintendent:**

1. Rewrite [FCPS Regulation 3250](#) to include:
   a. All offerings of the Advanced Mathematics program at the elementary level must include all extended indicators;
   b. While a variety of factors may influence decisions for placing elementary students in Advanced Mathematics, quantitative aptitude must be used as one of the primary determiners;
   c. The default placement for elementary students assessed as above average in quantitative aptitude will be Advanced Mathematics, regardless of whether they have been identified to receive Level IV Advanced Academic Services.

2. Remove AP Calculus AB as a course offering at TJHSST.
3. Modify the mathematics curriculum to include:
   a. Increasing the focus on students’ development of conceptual knowledge and standards of mathematical practices (e.g., make sense of problems and persevere in solving them);
   b. Raising the “floor” of the FCPS mathematics curriculum by requiring instruction in the standard curriculum to include all the current extended indicators and expanding the curriculum to include new and more rigorous extensions for advanced learners.

4. Identify the critical indicators missed when students accelerate at each grade level (3-8) and provide teachers with supplemental strategies to backfill instruction for students who accelerate.

5. Develop and execute a strategy to promote a common understanding and acceptance among parents, students, teachers and central office staff of students’ productive struggle as an extension of persistence and resilience.

6. Monitor and ensure division expectations are being followed at all schools for providing students with opportunities for rigor and acceleration in mathematics.
APPENDIX A

FCPS MATHEMATICS CURRICULUM STUDY

METHODOLOGY AND INSTRUMENTS
Appendix A

This appendix provides detailed information about the methodology used for the study.

Study Approach

The Office of Program Evaluation (OPE) typically uses a team approach to design evaluations and studies for Fairfax County Public Schools (FCPS). Teams generally comprise eight to ten people including the OPE director, manager and evaluators, and additional Central Office and school-based personnel selected to represent a variety of stakeholder perspectives regarding the study. The purpose of the team is to provide input at critical stages during the study. See Table A-1 for a description of the stages and team member roles. All study team members are not necessarily expected to participate in every stage. Requests for input are based on team member expertise.

Table A-1
Description of Study Team Roles

<table>
<thead>
<tr>
<th>Stage</th>
<th>Evaluation Team Member Role</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td>Provide information regarding informational needs, feedback on draft study questions, potential data sources, and timelines for data collection.</td>
</tr>
<tr>
<td>Instrumentation</td>
<td>Provide feedback on the scope and reasonableness of study instruments (surveys, interview/focus group protocols, additional student measures when appropriate).</td>
</tr>
<tr>
<td>Data Collection</td>
<td>Provide feedback on timing and communication mechanisms used to collect data using selected instruments.</td>
</tr>
<tr>
<td>Data Analysis and Synthesis</td>
<td>Provide feedback on the reasonableness of the conclusions based on data, particularly in cases of mixed results.</td>
</tr>
<tr>
<td>Report Writing and Presentation</td>
<td>Provide feedback on written reports and presentations, with particular sensitivity to clarity of language, utility of the information in meeting informational needs, and sensitizing OPE to any “political hot spots.”</td>
</tr>
</tbody>
</table>

In addition to six staff members in the Office of Program Evaluation, the evaluation team for the Math Sequence Study program included:

Craig Herring, director, Instructional Services Department  
Evan Glazer, principal, Thomas Jefferson High School Science Technology (THJSST)  
Rose Moore, math coordinator, Instructional Services Department  
Jennifer Allard, teacher, THJSST
Sampling

This study relied upon data from the entire division, when possible, and used a sampling approach when not possible. More specifically, to decrease data collection burden on schools, the study sampled 41 schools for collection of survey data from teachers, students, and parents and for detailed information on school practices and procedures related to mathematics instruction. When selecting the sampled schools, consideration was given to each pyramid with respect to the demographics of the school (i.e., Free and Reduced Meal status), presence of Advanced Academic Center (elementary and middle school only), and Advanced Placement (AP) or International Baccalaureate (IB) offering (high school only). The sample of schools was selected to represent the entire school division and allow for generalizing about FCPS as a whole.

Study Data Sources

The Mathematics Curriculum Study Design lists data collection sources and timelines for the Mathematics in FCPS Study. This appendix describes the methodology used to collect data from each of the data sources cited in the evaluation design.

Student Performance Data

To assess how the mathematics curriculum may impact students’ achievement, analyses were conducted using performance data from a variety of sources. Student Performance data included Algebra I and Algebra II Standards of Learning (SOL) test scores from school years 2010-11 through 2012-13, AB/IB, PSAT and SAT/ACT test scores.

Course Enrollment

Enrollment data gathered from the Department of Information Technology included course enrollment information for school year 2012-13 students and compacted math and advanced math flags for participation during elementary school. These data were used by OPE staff to describe both the students’ acceleration (whether they were at the typical grade level when taking a course, ahead of schedule, or behind schedule) and the rigor the student was exposed to during mathematics instruction (e.g., grade level or Advanced Math at the elementary level; Standard, Honors, or AP/IB courses at the middle and high school levels). Additionally, OPE staff constructed mathematics pathways for students that historically followed the acceleration and rigor students had experienced from elementary school through high school.

Teacher Focus Groups

In Fall 2013, OPE evaluators invited approximately 60 teachers across the division to participate in focus groups. The focus groups were organized by type of course and level. Teachers from the following courses were included: Elementary Advanced Math at 6th grade, middle school Math 7 Honors and Algebra I/Algebra I Honors, High School Algebra I and Algebra II/Algebra II Honors, TJHSST Algebra II Trigonometry Honors and Precalculus Honors. Focus groups were facilitated by OPE evaluators and were guided by a semi-structured protocol, including seven items, to gauge teachers’ expectations for student performance and student preparation. Teachers were compensated for their after-hours participation.
Stakeholder Surveys

In November and December 2013, surveys were distributed to select FCPS staff, parents, and students from 41 sampled schools across the division. All parents (from whom e-mail addresses were obtained) and a sample of students and teachers of various mathematics courses in the 41 sampled schools were asked to complete the respective stakeholder survey. The survey asked questions about stakeholder experiences and perceptions of the mathematics curriculum in FCPS. Overall, there were 4,755 parents, 189 teachers, and 2,947 students who completed the surveys.

- **Parent Survey:** All parents at the 41 sampled schools were asked to respond to an online survey during December 2013, through Survey Monkey. The survey was also available as a scannable paper survey and was available in seven languages. As seen in Table A-2, there were 1,737 parents at the elementary level, 1,131 parents at the middle school level, and 1,887 parents at the high school level who responded to the survey. The survey included a total of 17 items, focusing on general beliefs about the FCPS math curricula. Due to the multiple ways in which parents were contacted to participate in this study, families with multiple children in the sampled schools, etc., it is not possible to calculate a response rate for this survey. Parent responses represented all 41 schools participating in the survey.

<table>
<thead>
<tr>
<th>School Level</th>
<th>Number Responding to Survey</th>
<th>Percent of Parent Survey Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary</td>
<td>1,737</td>
<td>37%</td>
</tr>
<tr>
<td>Middle</td>
<td>1,131</td>
<td>24%</td>
</tr>
<tr>
<td>High</td>
<td>1,887</td>
<td>40%</td>
</tr>
</tbody>
</table>

- **Student Survey:** Students in Grade 5, Math 8, Algebra I and II, Geometry, and Precalculus from all 41 sampled schools were invited to take part in the student survey during December 2013. Surveys were administered either through Survey Monkey or scannable paper surveys, depending on school preference. The elementary student survey included 37 items; the middle school survey had 38 items; and the high school survey included 39 items on students’ general beliefs about the FCPS math curricula. A total of 120 classes in the 41 sampled schools were asked to and completed surveys. As observed in Table A-3, this included 1,078 students at the elementary level, 836 students at the middle school level, and 1,033 students at the high school level, for a total of 2,947 students who responded to the survey. Thus, an average of approximately 25 students from each of the 120 participating classrooms completed the survey.

<table>
<thead>
<tr>
<th>School Level</th>
<th>Number Responding to Survey</th>
<th>Percent of Student Survey Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary</td>
<td>1,078</td>
<td>37%</td>
</tr>
<tr>
<td>Middle</td>
<td>836</td>
<td>28%</td>
</tr>
<tr>
<td>High</td>
<td>1,033</td>
<td>35%</td>
</tr>
</tbody>
</table>

5Percentages may not total 100 due to rounding.
• **Teacher Survey**: Selected teachers from all 41 sampled schools were invited to take part in a teacher survey in November 2013, through Survey Monkey. There were 20 items on the web-based survey that asked teachers of Grade 5, Math 8, Algebra I, Algebra II, and Precalculus students their general beliefs about the FCPS math curricula. There was one open-ended question that invited teachers to describe the biggest challenge for students to master the math Program of Studies. Table A-4 provides information about the number of teachers from each math course who responded to the survey. At the middle and high school levels, teachers who taught multiple courses of interest were randomly assigned to report on a specific course. There were 189 teachers who completed the survey, which reflected a 63 percent response rate.

<table>
<thead>
<tr>
<th>Math Course</th>
<th>Number Responding to Survey</th>
<th>Percent of Teacher Survey Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 5</td>
<td>71</td>
<td>38%</td>
</tr>
<tr>
<td>Math 8</td>
<td>19</td>
<td>10%</td>
</tr>
<tr>
<td>Algebra I</td>
<td>5</td>
<td>3%</td>
</tr>
<tr>
<td>Algebra I Honors</td>
<td>12</td>
<td>6%</td>
</tr>
<tr>
<td>Geometry Honors</td>
<td>4</td>
<td>2%</td>
</tr>
<tr>
<td>Algebra II</td>
<td>27</td>
<td>14%</td>
</tr>
<tr>
<td>Algebra II Honors</td>
<td>19</td>
<td>10%</td>
</tr>
<tr>
<td>Precalculus</td>
<td>15</td>
<td>8%</td>
</tr>
<tr>
<td>Precalculus Honors</td>
<td>17</td>
<td>9%</td>
</tr>
</tbody>
</table>

**Profile of School Practices and Procedures Related to Mathematics Instruction**

All 41 sampled schools were invited to take part in a school profile survey during November 2013, through Survey Monkey. The survey included ten closed-ended items and four open-ended questions. The closed-ended items were on decision-making practices currently implemented regarding the math program and perceptions of the general and advanced math curricula. The open-ended items invited respondents to provide information on the most important challenges schools have experienced in relation to implementing the math program policies, issues raised by the school community, and any additional comments, questions, and concerns about the math curriculum and practices that may influence how students access the curricula. Respondents were asked to provide documentation regarding placement procedures. Documentation was received from five elementary, two middle schools, and four high schools. Forty of the 41 sampled schools completed the survey online, and one sampled school experienced difficulty submitting responses online and provided responses via e-mail. Thus, the response rate on the profile was 100 percent.

**Curriculum Documents**

OPE gathered the following curriculum documents directly from FCPS’ eCART library of resources:

- Program of Studies
- Pacing guides
- At a Glance documents for each grade level from K through AP Calculus
- Elementary and Secondary math sequence documents.
OPE requested and received the following curriculum documents from TJHSST:

- Algebra II Trigonometry Honors
- Precalculus Honors
- Advanced Placement Calculus BC

Review of the FCPS Mathematics Curriculum

In Winter 2013, the Department of Professional Learning and Accountability’s OPE contracted with an expert consultant to review the content, cognitive demand, and sequencing of the FCPS Mathematics POS. The document produced by the consultant was used by OPE as the primary source of evidence for studying the relevance, rigor, and sequencing of the FCPS mathematics curriculum. The consultant selected for this curriculum review was Curriculum Management Systems, Inc. (CMSi). CMSi and their founder, Fenwick W. English, developed the audit process captured in *The Curriculum Management Audit: Improving School Quality*, establishing their reputation as experts in the field of education. (Frase, English, & Poston, Jr., 2000). CMSi conducted a curriculum audit that involved an independent examination of documents which consisted of written curriculum guides developed by district personnel to guide instruction at all levels of the system. More specifically, the curriculum, prerequisites, and scope and sequence for success in Algebra II, Precalculus, and all of the courses at Thomas Jefferson High School for Science and Technology (TJHSST) were reviewed. Other documents included external curriculum sources such as the Common Core State Standards in mathematics, College and Career Readiness Standards from multiple sources, and mathematics standards from at least one international source (Finland). The analyses focused on the alignment of the content and rigor of the FCPS Mathematics POS to national standards, international standards, and standards for college and career readiness and relied upon the curriculum from elementary school through Precalculus/Precalculus Honors to address FCPS’ questions.

Documents and Artifacts from the FCPS Mathematics Office

Prior to the Central Mathematics Office interview, OPE requested documentation from ISD to help the study team understand current procedures in place in FCPS related to the math program. Documentation was requested on the following related topics:

- Processes used to write (and revise) the FCPS mathematics POS (i.e., how do you write a POS reflecting what we receive from the Virginia Department of Education, revise the POS as new standards are rolled in, decide whether a standard should be expected rather than extended).
- Guidance provided to schools at each level (elementary, middle, and high) on placement/acceleration of students and differentiation of curriculum (including EMIS).
- Communication with schools (e.g., memos, PowerPoint presentations, documents, etc.) related to math instruction, placement, acceleration, and differentiation in school years 2010-11, 2011-12, 2012-13, and 2013-14 (no need to include information related to online textbook adoption unless it covers other issues, too).
- Communication with the FCPS leadership team or School Board (i.e., response to School Board questions, memos, PowerPoint presentations, other documents) related to math instruction, placement, acceleration, and differentiation (no need to include SAG presentations or information related to online textbook adoption).
- List of FCPS Policies and Regulations that inform math instruction, acceleration, placement and/or any other math-related issues.
Interview with Central Office Staff Overseeing Mathematics

In Fall 2013, OPE evaluators conducted a semi-structured interview with ISD staff members to obtain information about the math program. To this end, an ISD program manager and director responded to six questions on placement, acceleration, differentiation, equity, and student preparation for the rigors of Algebra II.

Interview with TJHSST Staff

In April 2014, OPE evaluators conducted a semi-structured interview with THJSST staff to obtain information on the change in mathematics course sequence that the school is planning to implement for SY 2014-15. Toward this end, the principal, Math and Computer Science division head, and division head/teacher responded to two questions on the changes to TJHSST coursework.
APPENDIX B

FCPS MATHEMATICS CURRICULUM STUDY

DEFINITION OF TERMS
## Appendix B: Definition of Terms

### Table B-1
Definition of Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Procedural Knowledge</strong></td>
<td>Knowing <strong>how</strong> to do something. Procedural knowledge refers to an understanding of the steps or actions needed to solve mathematical problems. For example, knowing the steps to follow to carry out multi-digit multiplication is procedural knowledge. Knowing how to use formal mathematical language, such as writing equations, is also part of procedural knowledge. This term appears primarily in Report 3, Analysis of Curriculum Rigor and Challenge in the Mathematics Program.</td>
</tr>
<tr>
<td><strong>Conceptual Knowledge</strong></td>
<td>Knowing <strong>why</strong> something happens. Conceptual knowledge refers to understanding of the principles, rules, and generalizations that govern mathematics and the interrelationships between them. For example, understanding that when you add two positive numbers you should end up with a value that is larger than either of the original numbers would reflect conceptual knowledge in mathematics. This term appears primarily in Report 3, Analysis of Curriculum Rigor and Challenge in the Mathematics Program.</td>
</tr>
<tr>
<td><strong>Challenge</strong></td>
<td>Amount of academic (cognitive) demand <strong>experienced</strong> by individual students in response to instructional delivery of the written curriculum; students in the same class or course may experience different levels of challenge, depending upon prior knowledge, motivation, instruction, etc. This term appears primarily in Report 3, Analysis of Curriculum Rigor and Challenge in the Mathematics Program.</td>
</tr>
<tr>
<td><strong>Rigor</strong></td>
<td>Amount of academic (cognitive) demand <strong>designed</strong> into the written curriculum for each level of course; demand can span a range from low complexity (student mainly recalling and recognizing previously learned concepts and principles) to high complexity (student mainly using abstract reasoning, planning, analysis, judgment and creative thought). For this study, rigor is identified by the level of mathematics class or course (see additional explanation about grade-level math, advanced math, etc. in the bulleted rows below). This term appears primarily in Report 3, Analysis of Curriculum Rigor and Challenge in the Mathematics Program.</td>
</tr>
<tr>
<td><strong>Grade-level Math</strong></td>
<td>Elementary mathematics offering with an expected level of rigor described in the Program of Studies for that specific grade level; such as Grade 3 Math, Grade 4 Math, etc.</td>
</tr>
<tr>
<td><strong>Advanced Math</strong></td>
<td>Current FCPS approach to offering advanced mathematics at the elementary level (begun in SY 2011-12), such as Grade 3 Advanced Math, Grade 4 Advanced Math, etc.; this approach intends to allow students to be instructed at the level that is most appropriate for them through differentiation by learning grade level and above grade level standards. This approach also accelerates students by one grade level and provides greater rigor by incorporating enriched (known in FCPS as &quot;extended&quot;) standards.</td>
</tr>
<tr>
<td><strong>Compacted Math</strong></td>
<td>Former FCPS approach to offering advanced mathematics at the elementary level (last offered in SY 2010-11); typically accelerates students by one grade level.</td>
</tr>
<tr>
<td><strong>Standard Mathematics Courses</strong></td>
<td>Mathematics course offerings at the middle and high school levels (Grade 7 through Precalculus) with a typical level of rigor, such as Mathematics 7, Algebra I, Algebra II, etc.</td>
</tr>
<tr>
<td><strong>Honors Courses</strong></td>
<td>More rigorous mathematics course offerings at the middle and high school levels (Grade 7 through Precalculus) such as Mathematics 7 Honors, Algebra I Honors, Algebra II Honors, etc.</td>
</tr>
</tbody>
</table>
### Term Definition

**Acceleration** Content-based progression through a mathematics program at rates faster or at ages younger than expected; the general expectation from Virginia Department of Education is that students begin high school mathematics content (beginning with Algebra I) in grade 9.  

- **Hyperaccelerated** Students progressing through a mathematical program approximately two or more years faster than the expected; e.g., taking Algebra I in grade 7 instead of the expected grade 9.
- **Accelerated** Students progressing through a mathematical program approximately one year faster than expected; e.g. taking Algebra I in grade 8 instead of the expected grade 9.
- **Regular** Nonaccelerated; students progressing through a mathematics program at expected grade levels; e.g., taking Algebra I in grade 9, Geometry in grade 10, Algebra II in grade 11.
- **Late** Students progressing through a mathematical program approximately one year slower than the expected pattern; e.g., taking Algebra I in grade 10.

### Table B-2
**Definition of Courses Taken Along Acceleration Pathways**

<table>
<thead>
<tr>
<th>Pathway (Acceleration/Rigor)</th>
<th>Grades 3-6</th>
<th>Grade 7</th>
<th>Grade 8</th>
<th>Grade 9</th>
<th>Grade 10</th>
<th>Grade 11</th>
<th>Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Hyperaccelerated/Enriched)</td>
<td>Advanced Mathematics&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Algebra I Honors</td>
<td>Geometry Honors</td>
<td>Algebra II Honors</td>
<td>Precalculus Honors</td>
<td>AP/IB (or beyond)</td>
<td></td>
</tr>
<tr>
<td>B (Accelerated/Enriched)</td>
<td>Advanced Mathematics&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Mathematics 7 Honors&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Algebra I Honors</td>
<td>Geometry Honors</td>
<td>Algebra II Honors</td>
<td>Precalculus Honors</td>
<td>AP/IB</td>
</tr>
<tr>
<td>C (Accelerated/Enriched)</td>
<td>Mathematics</td>
<td>Mathematics 7 Honors&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Algebra I Honors</td>
<td>Geometry Honors</td>
<td>Algebra II Honors</td>
<td>Precalculus Honors</td>
<td>AP/IB</td>
</tr>
<tr>
<td>D (Accelerated/Standard)</td>
<td>Mathematics</td>
<td>Mathematics 7</td>
<td>Algebra I Honors</td>
<td>Geometry</td>
<td>Algebra II</td>
<td>Precalculus</td>
<td>AP/IB (or none)</td>
</tr>
<tr>
<td>E (Regular/Standard)</td>
<td>Mathematics</td>
<td>Mathematics 7</td>
<td>Mathematics 8</td>
<td>Algebra I, Part I</td>
<td>Algebra I</td>
<td>Geometry</td>
<td>Algebra II&lt;sup&gt;e&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> Curriculum for Advanced Mathematics in Grades 3 through 6 is equivalent to FCPS’ Mathematics POS with extended indicators for the next grade. For example, Grade 3 Advanced Mathematics covers FCPS’ Grade 4 Mathematics POS with extended indicators. Grade 6 Advanced Mathematics covers FCPS’ Grade 7 Mathematics POS with extended indicators.

<sup>b</sup> Curriculum for Mathematics 7 Honors is equivalent to FCPS’ Grade 8 Mathematics POS with extended indicators.

<sup>c</sup> Some FCPS students enroll in Standard level Algebra I while in Grade 8, but Algebra I Honors is more common.

<sup>d</sup> Students at some FCPS high schools have other course options, such as Probability and Statistics, Trigonometry, etc.

<sup>e</sup> Some FCPS students will enroll in Algebra II in Grade 12 because they have previously failed a high school-level mathematics course or because they took Geometry, Part I, during high school, rather than because they took Algebra I, Part I in ninth grade.

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<sup>6</sup> See Table B-2 for more details on the typical acceleration pathways for FCPS students.